MATH 111*i* WORKSHEET 8

Name: Solutions

Work in groups to answer as many problems as you can. Ask questions if you get stuck. The numbers used on this worksheet may require a calculator. Keep in mind that numbers you will have on exams will be nice enough to do without a calculator.

$\log_a(xy) = \log_a(x) + \log_a(y)$	$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$	$\log_a(1) = 0$	$\log_a(x^n) = n\log_a(x)$

1. Calculate the following:

(a) $\log_5(25) =$ Answer: 2

(b) $\log_3(1) =$ Answer: 0

(c) $\log_{16} 4 =$ Answer: 1/2

(d) $\log_5(1) =$ Answer: 0

(e) $\log_2(8) =$ Answer: 3

(f) $\log_7\left(\frac{1}{7}\right) =$ Answer: -1

(g) $\log_2(32) =$ Answer: 5

(h) $\log_2(32) =$ Answer: 5

(i) $\log_3\left(\frac{1}{9}\right) =$ Answer: -2

2. Find the value of a.

(a) $\log_a(64) = 6$ Answer: 2

(b) $\log_9(a) = -\frac{1}{2}$ Answer: $\frac{1}{3}$

(c) $\log_1\left(\frac{1}{8}\right) = a$ Answer: $-3/2$

(d) $\log_9\left(\frac{1}{81}\right) = a$ Answer: -2

(e) $\log_7(a) = 3$ Answer: 343

(f) $\log_a(36) = 2$ Answer: 6

3. Write the following expressions in terms of logs of x, y and z. (Note the bases are not specified as the laws work for any base)

(a) $\log(x^2y)$

$= \log(x^2) + \log(y)$

Answer: $2\log(x) + \log(y)$

(b) $\log\left(\frac{x^3y^2}{z}\right)$

$= \log(x^3y^2) - \log(z)$

$= \log(x^3) + \log(y^2) - \log(z)$

Answer: $3\log(x) + 2\log(y) - \log(z)$

(c) $\log\left(\frac{\sqrt{x}\sqrt[3]{y^2}}{z^4}\right)$

$= \log(\sqrt{x} \cdot \sqrt[3]{y^2}) - \log(z^4)$

$= \log(\sqrt{x}) + \log(\sqrt[3]{y^2}) - \log(z^4)$

$= \log(x^{1/2}) + \log(y^{2/3}) - \log(z^4)$

Answer: $\frac{1}{2}\log(x) + \frac{2}{3}\log(y) - 4\log(z)$

(d) $\log(xyz)$

$= \log(xy) + \log(z)$

$= \log(x) + \log(y) + \log(z)$

Answer: $\log(x) + \log(y) + \log(z)$

(e) $\log\left(\frac{x}{yz}\right)$

$= \log(x) - \log(yz)$

$= \log(x) - (\log(y) + \log(z))$

Answer: $\log(x) - \log(y) - \log(z)$

(f) $\log\left(\left(\frac{x}{y}\right)^2\right)$

$= 2\log(x/y)$

$= 2(\log(x) - \log(y))$

Answer: $2\log(x) + 2\log(y)$

(g) $\log\left((x)^{\frac{1}{3}}\right)$

Answer: $\frac{1}{3}\log(x)$

(h) $\log(x\sqrt{z})$

$= \log(x) + \log(\sqrt{z})$

$= \log(x) + \log(z^{1/2})$

Answer: $\log(x) + \frac{1}{2}\log(z)$

Cont.

(i) $\log\left(\frac{\sqrt[3]{x}}{\sqrt[3]{yz}}\right)$

$= \log(\sqrt[3]{x}) - \log(\sqrt[3]{yz})$

$= \log(x^{1/3}) - (\log(y^{1/3}) + \log(z^{1/3}))$

Answer: $\frac{1}{3}\log(x) - \frac{1}{3}\log(y) - \frac{1}{3}\log(z)$

(k) $\log\left(x\sqrt{\frac{\sqrt{x}}{z}}\right)$

$= \log(x) + \log\left(\left(\frac{\sqrt{x}}{z}\right)^{1/2}\right)$

$= \log(x) + \frac{1}{2}(\log(x^{1/2}) - \log(z))$

Answer: $\frac{5}{4}\log(x) + \frac{1}{2}\log(z)$

(j) $\log\left(\sqrt[4]{\frac{x^3y^2}{z^4}}\right)$

$= \frac{1}{4}\log\left(\frac{x^3y^2}{z^4}\right)$

$= \frac{1}{4}(\log(x^3y^2) - \log(z^4))$

$= \frac{1}{4}(\log(x^3) + \log(y^2) - \log(z^4))$

Answer: $\frac{3}{4}\log(x) + \frac{1}{2}\log(y) - \log(z)$

(l) $\log\left(\sqrt{\frac{xy^2}{z^8}}\right)$

$= \frac{1}{2}\log\left(\frac{xy^2}{z^8}\right)$

$= \frac{1}{2}(\log(xy^2) - \log(z^8))$

$= \frac{1}{2}(\log(x) + \log(y^2) - \log(z^8))$

Answer: $\frac{1}{2}\log(x) + \log(y) - 8\log(z)$

4. True or False? (Note the bases are not specified as the laws work for any base)

(a) $\log\left(\frac{x}{y^3}\right) = \log(x) - 3\log(y)$

$\log\left(\frac{x}{y^3}\right) = \log(x) - \log(y^3)$

$= \log(x) - 3\log(y)$

Answer: True

(b) $\log(a-b) = \log(a) - \log(b)$

$\log(a) - \log(b) = \log\left(\frac{a}{b}\right)$

$\neq \log(a-b)$

Answer: False

Cont.

(c) $\log\left(x^k\right) = k \cdot \log(x)$

Answer: True

(d) $\log(a) \cdot \log(b) = \log(a+b)$

Answer: False

(e) $\frac{\log(a)}{\log(b)} = \log(a-b)$

Answer: False

(f) $(\log(a))^k = k \cdot \log(a)$

Answer: False

(g) $\log_a\left(a^a\right) = a$

$\log_a(a^a) = a\log_a(a)$

$= a$

Answer: True

(h) $-\log\left(\frac{1}{x}\right) = \log(x)$

$-\log(1/x) = \log((1/x)^{-1})$

$= \log(x)$

Answer: True.

5. Combine the given expression into one single logarithm.

(a) $\log_2(A) + \log_2(B) - 2\log_2(C)$

$= \log_2(AB) - \log_2(C^2)$

Answer: $\log_2\left(\frac{AB}{C^2}\right)$

(b) $4\log_6(y) - \frac{1}{4}\log_6(z)$

$= \log_6(y^4) - \log_6(z^{1/4})$

Answer: $\log_6\left(\frac{y^4}{z^{1/4}}\right)$

Cont.

(c) $4\log_2(x) - \frac{1}{3}\log_2(x^2+1)$

$= \log_2(x^4) - \log_2((x^2+1)^{1/3})$

Answer: $\log_2\left(\frac{x^4}{(x^2+1)^{1/3}}\right)$

(d) $\log(5) + 2\log(x) + 3\log(x^2+5)$

$= \log(5) + \log(x^2) + \log((x^2+5)^3)$

Answer: $\log(5x^2(x^2+5)^3)$

(e) $3\log_2(A) + 2\log_2(B+1)$

$= \log_2(A^3) + \log_2((B+1)^2)$

Answer: $\log_2(A^3(B+1)^2)$

(f) $4\log_3(2x-1) - \frac{1}{2}\log_3((x+1)^2)$

$= \log_3((2x-1)^4) - \log_3(x+1)$

Answer: $\log_3\left(\frac{(2x-1)^4}{x+1}\right)$

(g) $2\log_8(x+1) + \log_8(x-1)$

$= \log_8((x+1)^2) + \log_8(x-1)$

Answer: $\log_8((x-1)(x+1)^2)$

(h) $\log_5(x^2-1) - \log_5(x-1)$

$= \log_5\left(\frac{x^2-1}{x-1}\right)$

Answer: $\log_5(x+1)$

(i) $\frac{1}{2}\log_4(y+1) - \frac{1}{2}\log_4(x-1)$

$= \log_4((y+1)^{1/2}) - \log_4((x-1)^{1/2})$

Answer: $\log_4\left(\left(\frac{y+1}{x-1}\right)^{1/2}\right)$

(j) $4\log(x) - \frac{1}{3}\log(x^2+1) + 2\log(x-1)$

$= \log(x^4) - \log((x^2+1)^{1/3}) + \log((x-1)^2)$

Answer: $\log\left(\frac{x^4(x-1)^2}{(x^2+1)^{1/3}}\right)$

Cont.

6. Solve the following logarithmic equations. (Note, if the base of the logarithm is not specified, it is not important in finding the solution)

(a) $\log(x) = -3$

oops

$\log_3(x) = -3$

Answer: $x = 1/27$

(b) $\log_{10}(3x - 2) = 2$

$\Rightarrow 3x - 2 = 100$

$\Rightarrow 3x = 102$

Answer: $x = 34$

(c) $2\log(x) = \log(2) + \log(3x - 4)$

$\Rightarrow \log(x^2) = \log(2(3x-4))$

$\Rightarrow x^2 = 2(3x-4)$

$\Rightarrow x^2 - 6x + 8 = 0$

$\Rightarrow (x-4)(x-2)$

Answer: $x = 2, 4$

(d) $\log(x) + \log(x - 1) = \log(4x)$

$\Rightarrow \log(x(x-1)) = \log(4x)$

$\Rightarrow x^2 - x = 4x$

$\Rightarrow x(x-5) = 0$

Answer: $x = 5$

(e) $\log_3(x + 25) - \log_3(x - 1) = 3$

$\log_3\left(\frac{x+25}{x-1}\right) = 3$

$\Rightarrow \frac{x+25}{x-1} = 27$

$\Rightarrow x + 25 = 27x - 27$

$\Rightarrow 52 = 26x$

Answer: $x = 2$

(f) $\log_9(x - 5) + \log_9(x + 3) = 1$

$\Rightarrow (x-5)(x+3) = 9$

$\Rightarrow x^2 - 2x - 24 = 0$

$\Rightarrow (x-6)(x+4) = 0$

$x = -4$ not in domain

$\Rightarrow x = 6$

Answer: $x = 6$.

(g) $\log(x) + \log(x - 3) = 1$

$\Rightarrow \log(x(x-3)) = 1$

$\Rightarrow$ oops

Need bases.

Answer:

(h) $\log_2(x - 2) + \log_2(x + 1) = 2$

$\Rightarrow \log_2((x-2)(x+1)) = 2$

$\Rightarrow (x-2)(x+1) = 4$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x-3)(x+2)$

Answer: $x = 3$

The End.